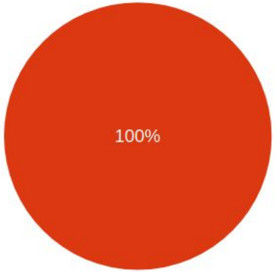
Coarse-graining Renormalization

6) Which of these could be a variable in a "macroeconomic" theory of Northern California?

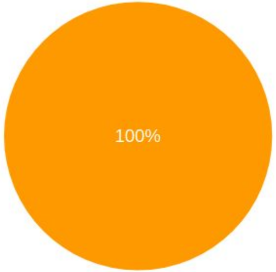
6 responses



- A. The employment history of the most recent hire to Google Research
- B. Total unemployment in Oakland, California
- C. The balance-sheet of a drycleaner's in Palo Alto
- D. The total biomass of all the Ph.D. candidates in the Stanford Computer Science d…

7) Which of these could function as a coarse-graining prescription?

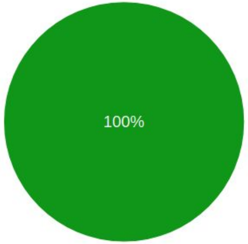
7 responses



- A. Mapping red to blue, and blue to red, in an RGB color image
- B. Anonymizing sensitive health data from a hospital, so that each person is represented b…
- C. Listing the frequencies of all the words in Tolstoy's War an…
- D. Translating a book from English to German

8) What needs to be true for the JPEG algorithm to count as a coarse-graining?

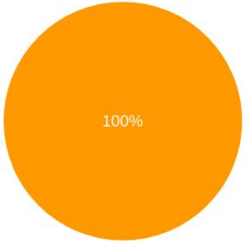
7 responses



- A. It reduces the file size: the output of the JPEG algorithm is smaller than the original bitmap image file.
- B. It is irreversible: given a JPEG compression, I can't recover the original image.
- C. It doesn't look the same as the original image.
- D. A and B, but not C

13) When beta is at the non-zero, but finite, fixed point, what happens when you coarse-grain:

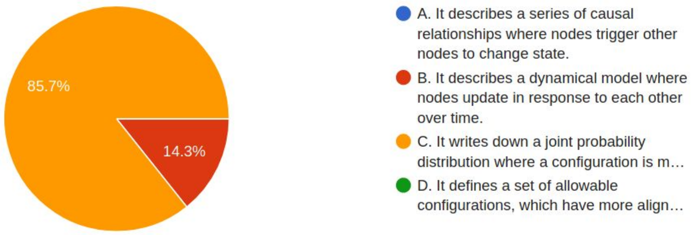
7 responses



- A. The little islands where nodes all share the same value get smaller and smaller, and it…
- B. Fluctuations away from a uniform +1 or uniform -1 state get smaller and smaller, and t…
- C. Little islands shrink, but there are larger islands of correlation that scale down to take their p…
- D. None of the above

1) The Ising model describes a system where neighbours tend to be found to have the same internal state. How does it do so?

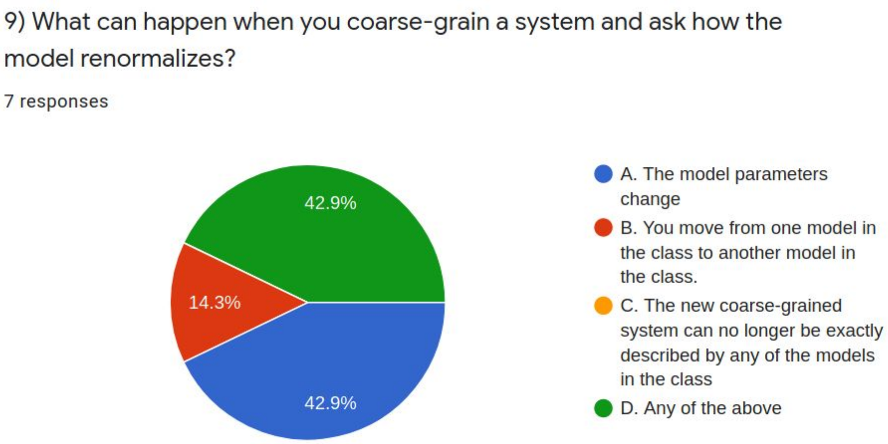
7 responses



Cohen, E. G. D. (2005). **Boltzmann and Einstein: Statistics and dynamics — An unsolved problem.** Pramana, 64(5), 635–643. doi:10.1007/BF02704573 (https://doi.org/10.1007/BF02704573)

While you can write an energy function or a *Hamiltonian* for the Ising Model, it does not mean that this is a dynamical model! Ising Model is an orthodox model in statistical physics which follows the idea of Boltzmann that dynamics can be followed by the probability distribution of the microscopic entities of the system at the equilibrium!

Einstein strongly disagreed with Boltzmann's approach to statistical mechanics, arguing that a statistical description of a system should be based on the dynamics of the system.
This opened the way, especially for complex systems, for other than Boltzmann statistics.



"non-renormalizable" theory:

A theory where when you coarse-grain, you get new terms that can't be neglected or fit into the original theory.

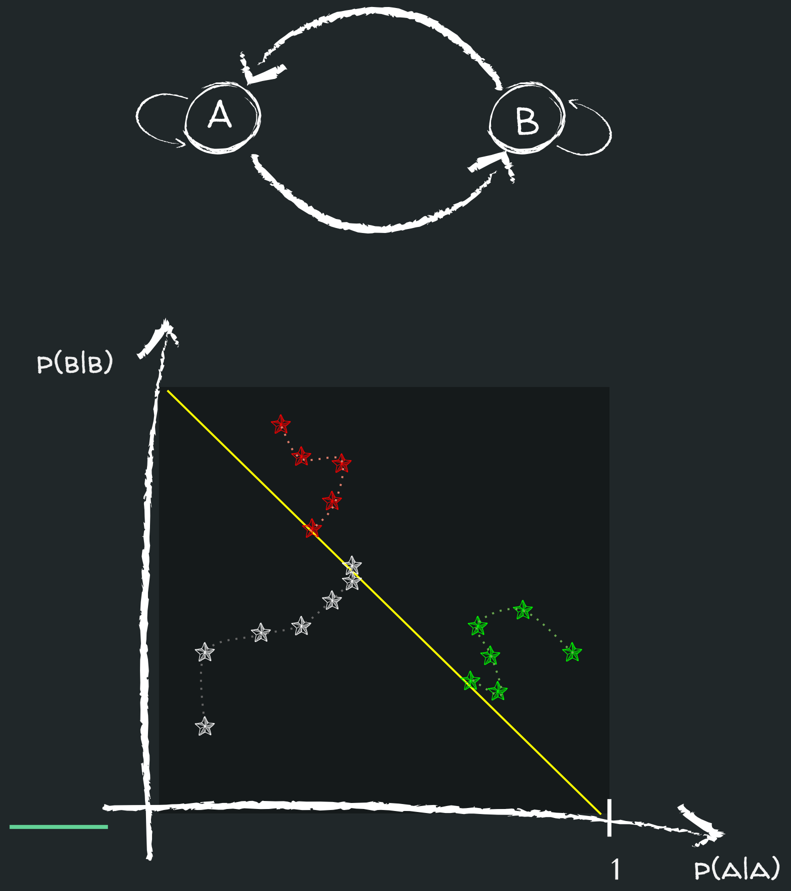
Whenever we are at the fixed points of RG transformation, model parameters do not change. An example is the stationary distribution of a Markov Chain after so many RG transformations. When we are talking about parameters "flow" in RG transformations we mean that how the parameters of a model change when you coarse-grain the data it's meant to describe. So, in a coarse-graining procedure, 1) the model parameters may change 2) or we may move from one model to another in the same class, say, in a cellular automata you can find projections that take you from one model (e.g., Rule 105) to another (e.g., Rule 150). See Israeli and Goldenfeld's work. Remember that we wanted to remain in the class of pairwise interactions for the Ising model.

3) In some cases, however, like in the case of Ising model, the coarse-grained system can no longer belong to the previous class (emergence of quartet interactions).

You can simply take a 2-state markov chain with different initial conditions and finally see how the parameters of the model flow in a 2d plane after each RG transformation.

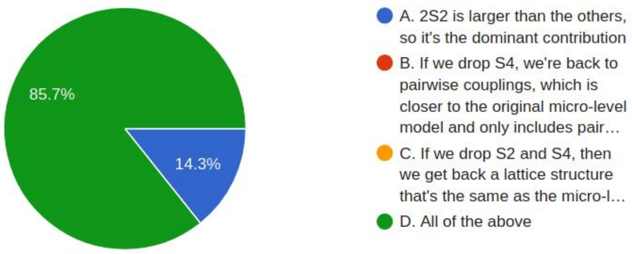
All the RG flows eventually lead to some manifold with lower dimension, here the yellow line!

See:
https://www.complexityexplorer.org/courses/67-introduction-to-renormalization



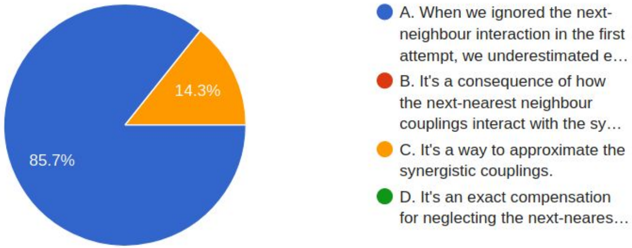
10) What's the justification for including the 2S2 coupling, but not the S2 coupling or the S4 coupling?

7 responses



11) What's the justification for changing the 2S2 coupling to 3S2?

7 responses



Answer: 10 (D), 11 (A)

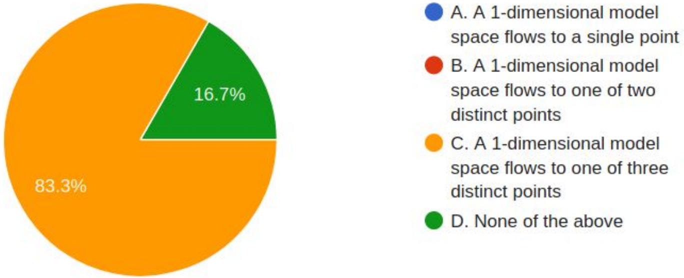
This is a pretty vicious approximation. We're dropping a huge amount of structure in the model just to keep it in the same model class.

This is a complete hack; we just by fiat approximate a lattice with these cross-wise nearest neighbour couplings as a square one. There's no good justification other than (as we'll see) it enables us to get closer to reality. It's definitely not the case that it enables us to handle the synergistic four-way couplings -- these are much stranger, and capture something invisible to the pairwise case.

Don't be too hard on this approximation. It was something people did a long time ago when they were really wrestling with how to handle models like this. Today we have much neater ways to go about these approximations, in particular, by going into Fourier space -- but that's another story. What I love about this account is that you really get a sense for how people hack away when they don't have an answer yet.

12) What's the renormalization group flow for the Ising model in our coarse-grained approximation?

6 responses

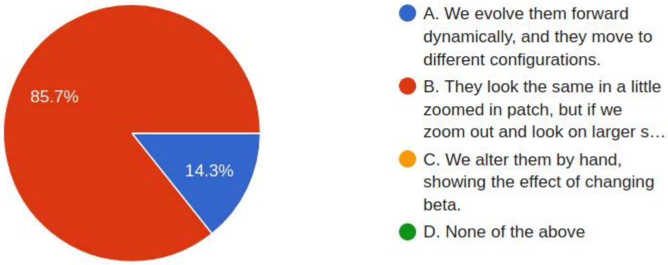


Answer: (C).

When beta is below a critical value, repeated renormalizations drive you to zero -- a decoupled state. When it's above a critical value, beta gets extremely large. When it's precisely at the critical value, it stays fixed at a finite (non-zero) value.

14) The three models in Douglas Ashton's simulation look pretty similar in terms of their correlations at first. What happens to make them look different?

7 responses



Answer: (B). In each of the three cases, Douglas has run a simulation and just taken one image -- basically, sampling the joint probability distribution for each of the three models.

There's no dynamical evolution shown here, and nobody is changing the models by hand to become more different. Locally, the models look very similar. But if we zoom out to larger scales -- i.e., if we repeatedly do a decimation transformation, dropping half of the points out and moving the others closer together (so more and more of the image can fit on the screen), we see that they're very different on large scales. The effective beta parameter goes either to zero (totally correlation), or to infinity (totally decorrelated), or (if precisely poised at the critical temperature) it remains the same, self-similar on all scales.